

# Phase Diagram of Vertically Shaken Granular Matter

Peter Eshuis,<sup>1</sup> Ko van der Weele,<sup>2</sup> Devaraj van der Meer,<sup>1</sup> Robert Bos,<sup>1</sup> and Detlef Lohse<sup>1</sup>

<sup>1</sup> *Physics of Fluids, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands*

<sup>2</sup> *Mathematics Department, Division of Applied Analysis, University of Patras, 26500 Patras, Greece*

(Dated: February 2, 2008)

A shallow, vertically shaken granular bed in a quasi 2-D container is studied experimentally yielding a wider variety of phenomena than in any previous study: (1) bouncing bed, (2) undulations, (3) granular Leidenfrost effect, (4) convection rolls, and (5) granular gas. These phenomena and the transitions between them are characterized by dimensionless control parameters and combined in a full experimental phase diagram.

PACS numbers: 05.65.+b, 45.70.-n, 45.70.Qj

## I. INTRODUCTION

Vertically shaken granular matter exhibits a wealth of fluid-like phenomena such as undulations [1, 2, 3], wave patterns [4, 5], granular Leidenfrost effect [6] and convection rolls [7]. However, while in normal fluids and gases these phenomena are fully understood, this is much less the case for their granular counterparts. In order to get a better understanding of the underlying physics, here we present an experimental overview of the various effects observed in a vibrated bed of glass beads, identifying the dimensionless control parameters that govern them. The main goal of the paper is to construct an experimental phase diagram in which all the observed phenomena are combined.

Our experimental setup (Fig. 1) consists of a quasi 2-D perspex container of dimensions  $L \times D \times H = 101 \times 5 \times 150$  mm (with  $L$  the container length,  $D$  the depth, and  $H$  the height), partially filled with glass beads of diameter  $d = 1.0$  mm, density  $\rho = 2600$  kg/m<sup>3</sup>, and coefficient of normal restitution  $e \approx 0.95$ . The setup is mounted on a sinusoidally vibrating shaker with tunable frequency  $f$  and amplitude  $a$ . Most of the experiments presented in this paper are performed by upsweep-experiments in which the frequency is linearly increased at 75 Hz/min. These experiments are recorded with a high-speed camera capturing 2000 frames per run; adequate recording times (4 to 16 seconds) are obtained by adjusting the frame rate.

The natural dimensionless control parameters to analyze the experiments are (i) the shaking parameter, or Froude number,  $a^2\omega^2/g\ell$  (with  $\omega = 2\pi f$  and  $g = 9.81$  m/s<sup>2</sup>), being the ratio of the kinetic energy inserted into the system by the vibrating bottom and the potential energy associated with a typical displacement of the particles  $\ell$ , (ii) the number of bead layers  $F$ , (iii) the inelasticity parameter  $\varepsilon = (1 - e^2)$ , and (iv) the aspect ratio  $L/h_0$ , where  $h_0$  denotes the bed height at rest. The parameter  $\varepsilon$  is taken to be constant in this paper, since we ignore the velocity dependence and use the same beads throughout. The aspect ratio varies by changing the bed height  $h_0$  (i.e., the number of layers  $F$ ) but remains large in all experiments,  $L/h_0 \gg 1$ . We will systematically vary the first two dimensionless parameters, by chang-

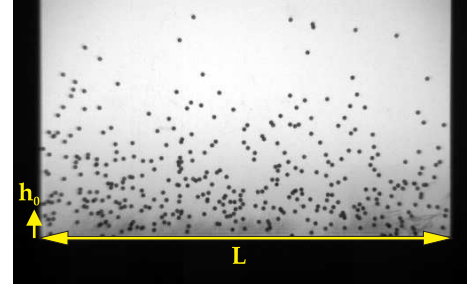


FIG. 1: The experimental setup in which glass beads of diameter  $d = 1.0$  mm are vibrofluidized. The length of the container is  $L = 101$  mm; the bed height at rest ( $h_0$ ) is varied in our experiments such that the aspect ratio  $L/h_0$  always remains large. The container depth is only five particle diameters, making the setup quasi two-dimensional.

ing the amplitude  $a$ , the frequency  $f$ , and the number of layers  $F$ .

The most intriguing of the four parameters above is the first one, the shaking parameter, since the typical displacement of the particles  $\ell$  is influenced in a non-trivial way by the vibration intensity and the number of particle layers. For *mild* fluidization the displacement of the particles is determined by the amplitude of shaking  $a$ , since the bed closely follows the motion of the bottom. The energy ratio in this case becomes identical to the well known dimensionless shaking acceleration:

$$\Gamma = \frac{a\omega^2}{g}. \quad (1)$$

For *strong* fluidization the particles no longer follow the bottom, so (instead of  $a$ ) some intrinsic length scale needs to be taken for  $\ell$ , such as the particle diameter  $d$ . This leads to the dimensionless shaking strength  $S$  [6]:

$$S = \frac{a^2\omega^2}{gd}. \quad (2)$$

At intermediate fluidization, we will encounter phenomena in which there is a competition of length scales. In this region the transitions are affected by changing one of the competing length scales, meaning that the choice of

the appropriate shaking parameter is not a priori clear. This will become an issue in particular for the transition from undulations to the granular Leidenfrost effect described in Section IV.

In the following Sections, the various phenomena observed in our system are discussed one by one, in the order in which they appear as the fluidization is increased: bouncing bed (Section II), undulations (III), granular Leidenfrost effect (IV), convection rolls (V), and granular gas (VI). Finally, in Section VII all five phenomena will be combined in a phase diagram of the relevant shaking parameter versus the number of layers.

## II. BOUNCING BED

For shaking accelerations  $\Gamma \leq 1$  (and even for  $\Gamma$  slightly above 1) the granular bed behaves as a solid, co-moving with the vibrating bottom and never detaching from it. In order to detach, the bottom must at some point during the cycle have a downward acceleration that overcomes gravity (as for a single bouncing ball [8, 9, 10, 11, 12, 13, 14, 15, 16]) *plus* the friction between the bed and the walls of the container. These walls carry a considerable portion of the bed weight, as described by the Rayleigh-Jansen model [17]. Once the detachment condition is fulfilled, the bed bounces in a similar way as a single particle would do: We call this a bouncing bed, see Fig. 2.

The value of  $\Gamma$  at which the transition from solid to bouncing bed occurs has been determined by gradually increasing the frequency  $f$  (for three fixed shaking amplitudes  $a = 2.0, 3.0$  and  $4.0$  mm). The onset value grows

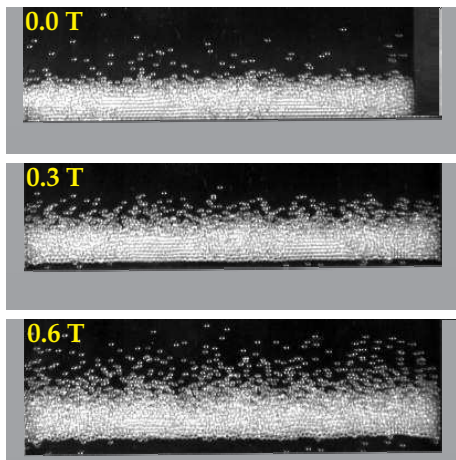


FIG. 2: Time-series of a bouncing bed for  $F = 8.1$  layers of  $d = 1.0$  mm glass beads at shaking acceleration  $\Gamma = 2.3$  ( $a = 4.0$  mm,  $f = 12.0$  Hz). The phase of the sinusoidally vibrating bottom is indicated in each snapshot, where  $T$  is the period of shaking ( $y_{bottom}(t) = a \sin(2\pi t/T)$ ). The friction between the particles and the container walls causes the downward curvature of the bed close to the sidewalls that is visible in the lower snapshot. [Enhanced online: link to movie of the bouncing bed.]

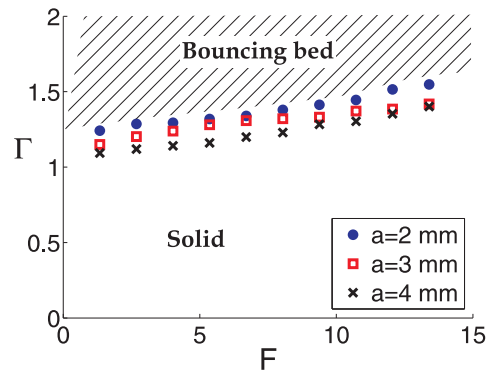


FIG. 3: (color online). The transition from solid behavior to bouncing bed is governed by the shaking parameter  $\Gamma$ . The critical value (here determined for three fixed amplitudes  $a = 2.0, 3.0, 4.0$  mm) increases roughly linearly with the number of particle layers  $F$ .

with the number of layers  $F$ , as shown in Fig. 3. The reason for this is the larger contact area with the front- and sidewalls causing a proportionally higher frictional force. Indeed, the onset value of  $\Gamma$  is seen to increase roughly linearly with the number of layers.

Figure 3 indicates that for the current transition (which occurs at mild fluidization)  $\Gamma$  is a good dimensionless parameter, as explained in the Introduction. It is not ideal, as exemplified by the fact that the onset values do not exactly coincide for the different amplitudes of shaking, but for a different choice of the shaking parameter ( $S$ ) the onset values differ much more.

## III. UNDULATIONS

Starting from a bouncing bed and increasing the shaking frequency  $f$ , three different phenomena are observed: (a) For  $F \leq 3$  layers the bed is vaporized and becomes a granular gas (Section VI), (b) for  $3 < F \leq 6$  convection rolls form (Section V), and (c) for  $F > 6$  layers the bed develops undulations (also called "arching" or "ripples" in the literature [1, 2, 3, 5, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]), which we will cover in this section.

In the undulations regime, the granular bed shows standing wave patterns similar to a vibrating string as shown in Fig. 4. The standing waves oscillate at twice the period of shaking, and are therefore also known as  $f/2$ -waves [4, 5]. The container (length  $L$ ) accommodates an integer number  $n$  of half wavelengths of the granular string:

$$L = n \frac{\lambda}{2}, \quad n = 1, 2, 3, \dots \quad (3)$$

where  $\lambda$  is the length of one arch in the undulation pattern. This  $\lambda$  represents a new length scale in the system

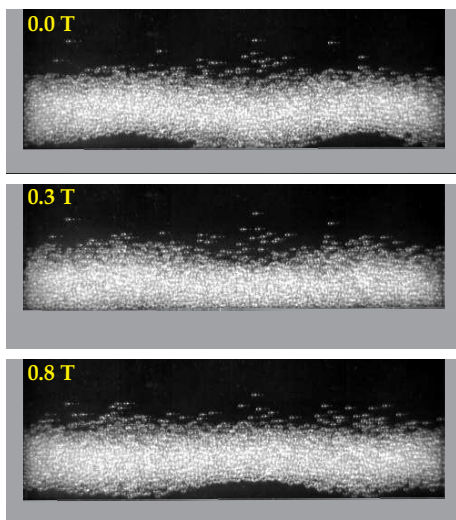


FIG. 4: One complete standing-wave cycle of the  $n = 4$  undulation mode for  $F = 9.4$  particle layers at  $\Gamma = 12$  ( $a = 2.0$  mm,  $f = 39.3$  Hz). The undulation cycle takes  $2/f$ , i.e., twice the period of shaking. [Enhanced online: link to movie of the  $n = 4$  undulation mode.]

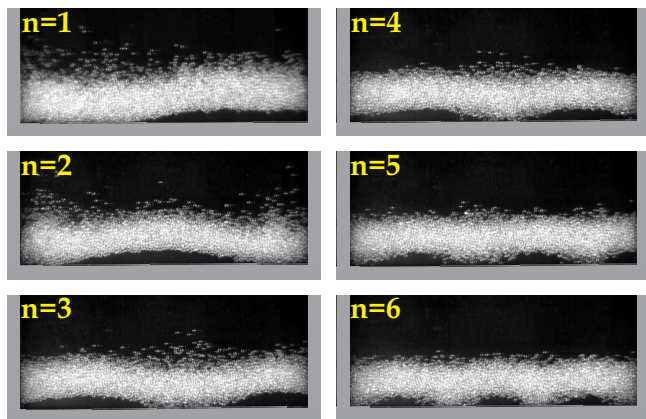


FIG. 5: Six successive undulation modes, for  $F = 9.4$  layers and  $a = 2.0$  mm, at shaking frequencies  $f = 29.0, 32.6, 38.2, 39.3, 46.1, 50.2$  Hz. The mode number  $n$  (the number of half-wavelengths fitting the container length  $L$ ) increases with the shaking intensity.

besides the shaking amplitude  $a$  and the particle diameter  $d$ . Unlike these previous length scales,  $\lambda$  is connected to the elastic properties of the particles, which play an important role in the undulations.

Generally the first undulation to be formed is the  $n = 1$  mode, and for increasing fluidization the higher modes depicted in Fig. 5 successively appear. They are triggered by the horizontal dilatancy the bed experiences when it collides with the vibrating bottom [3]: the string of particles along the bottom *dilates* and is forced to form an arch. Using this physical picture, Sano [3] was able to derive a theoretical form of the undulation modes, which qualitatively agrees with the form of the experimental

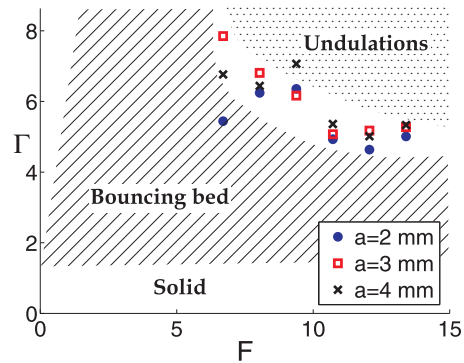


FIG. 6: (color online). The transition from bouncing bed to undulations in the  $(\Gamma, F)$ -plane, for three fixed values of the shaking amplitude ( $a = 2.0, 3.0, 4.0$  mm). The critical value of the shaking acceleration  $\Gamma$  decreases with growing number of particle layers  $F$ , since the horizontal dilation of the bottom layer (required to trigger undulations, see text) becomes more pronounced as a result of the extra layers on top.

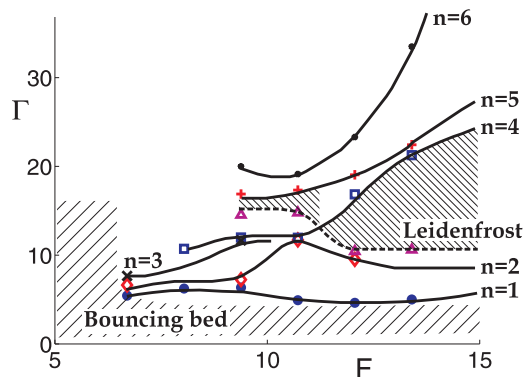


FIG. 7: (color online). The onset of the successive undulation modes  $n = 1, 2, \dots, 6$  at a fixed shaking amplitude  $a = 2.0$  mm. The mode number  $n$  increases with the shaking parameter  $\Gamma$ , but occasionally the undulations give way to the granular Leidenfrost effect (the hatched regions above the dashed curve), where a dense cluster without any arches is floating on a uniformly dilute granular layer.

ones in Fig. 5. We observe that each collision with the bottom causes a shock wave through the bed at a roughly constant speed of  $v = \lambda f = 2$  m/s. This sends compaction waves along the arch, starting out from the lower parts and meeting in the center. At this point the waves bring each other to a halt and the center falls down to the bottom. (At the same time, the previous lower parts are now elevated.) This occurs after one shaking period and the collision with the bottom generates new shock waves, repeating the series of events. It takes precisely two periods of shaking to complete one full oscillation of the undulation pattern.

First we focus on the transition from the bouncing bed behavior to the undulations, i.e., on the appearance of

the  $n = 1$  mode. In Fig. 6 this transition is shown in the  $(\Gamma, F)$ -phase diagram for three fixed amplitudes of shaking,  $a = 2.0, 3.0$  and  $4.0$  mm. We observe that the onset value of  $\Gamma$  decreases with growing number of layers  $F$ . The reason for this is that the necessary horizontal dilation (of the lower layer) upon impact with the bottom is more readily accomplished due to pressure from the extra layers on top.

It is seen in Fig. 6 that the data for the three different shaking amplitudes coincide reasonably, except at the threshold value of  $F = 6$  layers. Presumably, at this small value of  $F$  the dilation can only become sufficient if the density is locally enhanced by a statistical fluctuation; when the experiment would be repeated many times the agreement between the averaged data for various  $a$  is expected to become better. For  $F < 6$  layers no undulations are found, since the particle density is then definitely too small (even in the presence of fluctuations) to reach the required level of dilation.

Now we come to the higher undulation modes. Figure 7 displays the observed modes for shaking amplitude  $a = 2.0$  mm. As already observed in Fig. 5 the mode number  $n$  increases for growing  $\Gamma$ . However, the sequence of modes is seen to be interrupted somewhere in the middle: Here the undulation pattern gives way to the granular Leidenfrost state [6], in which a cluster of slow particles is floating on top of a dilute layer of fast particles. Normally, this state appears at the end of the undulation regime (see Section IV), but when a certain standing wave pattern is energetically unfavorable the system chooses the Leidenfrost state instead. In Fig. 7 we see that this happens to the  $n = 3$  undulation, which is completely skipped from the sequence for  $F \gtrsim 12$  layers. This can be understood from the fact that the  $n = 3$  mode has an antinode at the sidewall (i.e., a highly mobile region), whereas the friction with the wall tends to slow down the particles here. This inherent frustration gives rise to the appearance of the granular Leidenfrost effect.

Likewise, the small Leidenfrost region for  $9 \leq F \lesssim 12$  below the onset line of the  $n = 5$  undulation may well be the result of a frustrated  $n = 5$  mode here. The frustration is however not strong enough to skip the mode as in the  $n = 3$  case. In our experiments, we find that the intermediate regions of the Leidenfrost state become smaller for larger shaking amplitude  $a$ . For  $a = 4.0$  mm they have disappeared altogether from the undulation regime, as we will show in Section VII.

#### IV. GRANULAR LEIDENFROST EFFECT

When the shaking frequency is increased beyond a critical level, the highest undulation mode becomes unstable and we enter the granular Leidenfrost regime [6]: Here a dense cloud of particles is elevated and supported by a dilute gaseous layer of fast beads underneath, see Fig. 8. The bottom layer of the undulations is completely evap-

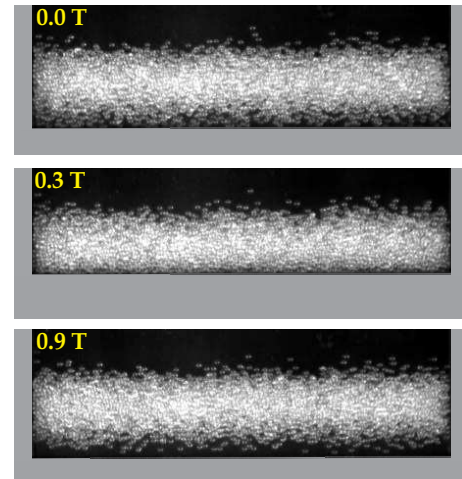


FIG. 8: Snapshots of the granular Leidenfrost effect for  $F = 8.1$  particle layers shaken at  $f = 43.0$  Hz and  $a = 3.0$  mm (corresponding to a dimensionless acceleration  $\Gamma = 22$  or shaking strength  $S = 67$ ). A dense cluster is elevated and supported by a dilute layer of fast particles underneath. The cluster never touches the vibrating bottom, which makes this state distinctively different from the bouncing bed or the undulations. [Enhanced online: link to movie of the granular Leidenfrost effect.]

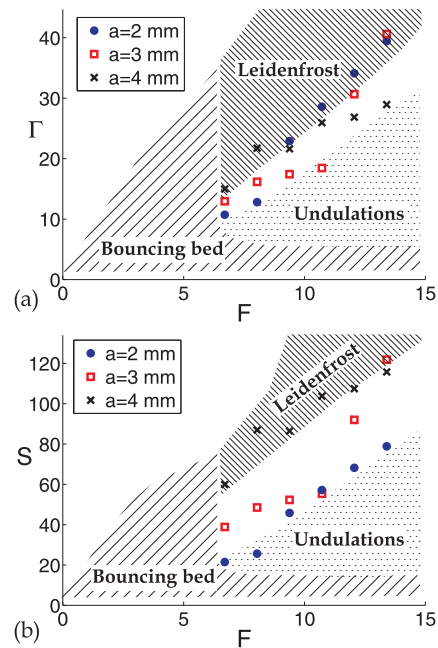


FIG. 9: (color online). The transition from undulations to the granular Leidenfrost effect for increasing frequency  $f$  and fixed amplitude  $a = 2.0, 3.0, 4.0$  mm: (a) In the  $(\Gamma, F)$ -plane, (b) in the  $(S, F)$ -plane. Since in our experiments the Leidenfrost state always originates from the undulation regime, the same minimum number of layers is needed:  $F > 6$ . The critical values of  $\Gamma$  and  $S$  increase with  $F$ , since a higher energy input is required to elevate a larger cluster.



orated and forms the gaseous region on which the cluster floats. The phenomenon is analogous to the original Leidenfrost effect in which a water droplet hovers over a hot plate on its own vapor layer, when the temperature of the plate exceeds a critical value [34]. The vaporized lower part of the drop provides a cushion to hover on, and strongly diminishes the heat contact between the plate and the drop, enabling it to survive for a relatively long time.

In Fig. 9 the transition from the undulations to the granular Leidenfrost state is shown, both in the  $(\Gamma, F)$  and in the  $(S, F)$ -plane. Despite the fact that we have left the mild fluidization regime behind,  $\Gamma$  still appears to be the governing shaking parameter, since the data for the different amplitudes ( $a = 2.0, 3.0, 4.0$  mm) collapse better on a single curve in the  $(\Gamma, F)$  than in the  $(S, F)$ -plane. In fact, the critical  $S$ -values in the latter plane show a systematic increase for growing amplitude  $a$ .

This is in contrast to the observations on the granular Leidenfrost effect in a previous study of smaller aspect ratio [6, 35], for  $d = 4.0$  mm glass beads in a 2-D container, where the phase transition was shown to be governed by the dimensionless shaking strength  $S$ . In that case the Leidenfrost state was reached directly from the solid, bouncing bed regime, without the intermediate stage of undulations. Presumably this was due to the much smaller aspect ratio  $L/h_0$ , which was in the order of 1 (against  $L/h_0 \sim 10$  in the present Leidenfrost experiments) [36]. Another important difference was that the depth of the setup was just slightly more than one particle diameter (against 5 diameters in the present setup), so the motion of the granular bed was much more restricted; indeed, the floating cluster in Ref. [6] showed a distinctly crystalline packing. It may be concluded, as already remarked in the Introduction, that the Leidenfrost effect lies in the regime of intermediate fluidization, where both  $\Gamma$  and  $S$  are candidates to describe the behavior of the granular bed. The proper choice of the shaking parameter here depends not only on the degree of fluidization, but also on the dimensions of the specific system investigated.

## V. CONVECTION ROLLS

In our experiments, granular convection rolls are formed at high fluidization from either (a) the bouncing bed (for  $3 < F \leq 6$  layers) or (b) the granular Leidenfrost effect (for  $F > 6$ ). In both cases the onset of convection is caused by a set of particles in the cluster that are more mobile (higher granular temperature) than the surrounding area, creating an opening in the bed. These particles have picked up an excess of energy from the vibrating bottom (due to a statistical fluctuation) and collectively move upwards, very much like the onset of Rayleigh-Bénard convection in a classical fluid heated from below [37, 38, 39, 40, 41, 42, 43]. This upward motion of the highly mobile beads must be balanced by a

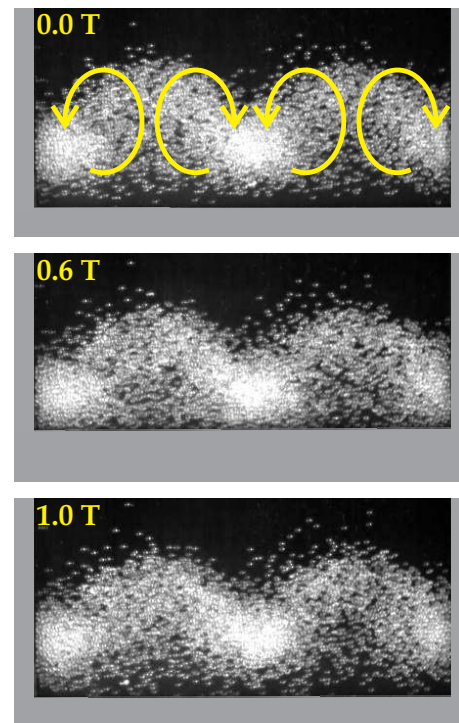


FIG. 10: Granular convection for  $F = 8.1$  layers at  $f = 73.0$  Hz and  $a = 3.0$  mm (dimensionless shaking strength  $S = 193$ ), showing four counter-rotating rolls. The beads move up in the dilute regions (high granular temperature) and are sprayed sideways to the three dense clusters (low granular temperature). In our system two clusters are always located near the sidewalls, which have a relatively low granular temperature due to the extra dissipation. [Enhanced online: link to movie of granular convection.]

downward movement of neighboring particles, leading to the formation of a convection roll.

The downward motion is most easily accomplished at the sidewalls, due to the extra source of dissipation (i.e., the friction with the walls), and for this reason the first convection roll is always initiated near one of the two sidewalls. Within a second, this first roll triggers the formation of rolls throughout the entire length of the container, leading to a fully developed convection pattern as in Fig. 10.

Granular convection has been studied extensively at mild fluidization [23, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69], for which the convection is principally boundary-driven. However, the buoyancy-driven convection observed here occurs at *high* fluidization and this has been reported much more rarely in the literature. We are aware of only one numerical study by Paolotti *et al.* [7] and here present the first experimental observations. In the numerical model by Paolotti *et al.* the container walls were taken to be perfectly elastic, leading to convection patterns in which the rolls were either moving up or down along the sidewalls, whereas in our system (with dissipa-

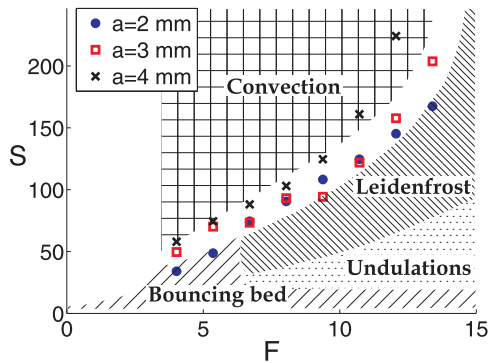


FIG. 11: (color online). The transition towards granular convection from the bouncing bed ( $3 < F \leq 6$ ) and the Leidenfrost state ( $F > 6$ ) in the  $(S, F)$ -plane, for fixed shaking amplitude  $a = 2.0, 3.0$  and  $4.0$  mm. Just as for the Leidenfrost transition, the convection sets in at higher values of  $S$  as the number of layers  $F$  is increased, because a higher dissipation must be overcome for larger bed heights.

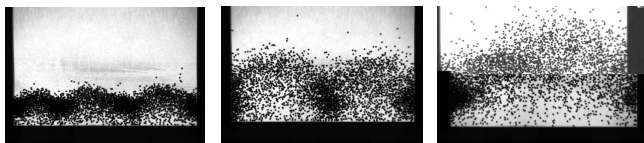


FIG. 12: Convection patterns for  $F = 18.8$  layers of  $1.0$  mm stainless steel beads at three consecutive shaking strengths:  $S = 58$  ( $a = 2.0$  mm,  $f = 60.0$  Hz),  $S = 130$  ( $a = 3.0$  mm,  $f = 60.0$  Hz), and  $S = 202$  ( $a = 4.0$  mm,  $f = 56.0$  Hz). For increasing  $S$  the convection rolls expand, hence a smaller number of them fits into the container. The steel beads behave qualitatively (but not quantitatively) the same as the glass beads used in the rest of the paper.

tive walls) they always move down.

Figure 11 shows the transition to convection in the  $(S, F)$ -plane, starting from either the bouncing bed or the Leidenfrost state, which are taken together because the transition dynamics is the same in both cases. This is the first instance that the data points (acquired for all shaking amplitudes  $a = 2.0, 3.0$  and  $4.0$  mm) collapse better for the shaking parameter  $S$  than for the dimensionless acceleration  $\Gamma$ , meaning that  $S$  is the preferred control parameter for the convection transition.

The onset values of  $S$  grow with the number of layers  $F$ , because for large  $F$  more energy input from the vibrating bottom is necessary to break through the larger dissipation in the granular bed and trigger the first convection roll. Related to this, the number of rolls in the convection pattern decreases for growing  $F$ : Due to the larger total dissipation, the dense clusters of each roll grow in size. Hence the convection rolls become wider, meaning that less rolls fit into the container.

When, for a given number of layers  $F$ , the shaking strength  $S$  is increased (either via the frequency  $f$  or the amplitude  $a$ ), the number of rolls in the convection pat-

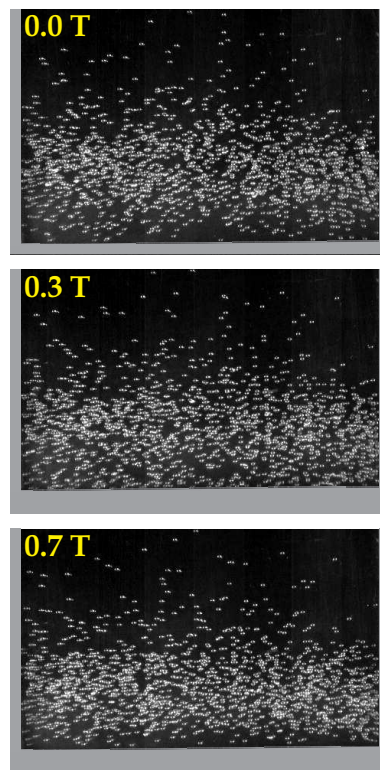


FIG. 13: Granular gas for  $F = 2.7$  layers at  $f = 50.0$  Hz and  $a = 3.0$  mm ( $\Gamma = 30$ ), which has originated from a bouncing bed by increasing the shaking beyond a critical level (see Fig. 14b). With the vibration power available in our system, granular gases are only observed for  $F \leq 3$  layers. [Enhanced online: link to movie of a granular gas.]

tern becomes smaller. This is illustrated in Fig. 12: The higher energy input induces expansion of the convection rolls, and the number of rolls decreases stepwise as  $S$  is increased. The steps involve two rolls at a time, since the pattern always contains an even number of rolls due to the downward motion imposed by the sidewalls.

## VI. GRANULAR GAS

In this section we briefly discuss the fifth and last phenomenon observed in our system: A granular gas, being a dilute cloud of particles moving randomly throughout the container as in Fig. 13 [70, 71, 72, 73, 74, 75, 76, 77, 78, 79]. This state is only observed for a small number of layers ( $F \leq 3$ ) and always originates from the bouncing bed regime. At these small  $F$ , the bed shows expansion and compaction during every vibration cycle due to the low total dissipation. At the critical value of the shaking parameter, the bed expands to such an extent that it evaporates and forms a gas.

The evaporation of the bouncing bed requires more energy as the number of layers  $F$  increases. The transition seems to be controlled by the shaking acceleration  $\Gamma$  (which also governs the transition from solid to bouncing

bed) rather than the shaking strength  $S$ . However, the data points available are too few ( $F \leq 3$ ) to make this conclusive. The measurements will be presented in the full phase diagram of the next section.

## VII. PHASE DIAGRAM

Finally, all the phenomena and associated transitions described in the previous sections are combined in the phase diagram of Fig. 14. Both shaking parameters ( $\Gamma$  and  $S$ ) are used in this diagram, each of them indicating the respective transitions they were found to govern. The parameter  $\Gamma$  is shown along the left vertical axis and the corresponding data points (the critical  $\Gamma$  values) are colored red. The parameter  $S$  is plotted along the right vertical axis and the corresponding experimental data are colored blue; this concerns only the  $+$ -signs at the convection transition [80]. For comparison the  $\Gamma$ -axis is kept the same in all three phase diagrams.

Figure 14 contains three separate phase diagrams for the three fixed shaking amplitudes we have used throughout the paper:  $a = 2.0, 3.0$ , and  $4.0$  mm. Most of the phase transitions are hardly affected, with the exception of the various transitions between the undulations and the Leidenfrost state. These transitions lie in the regime of *intermediate* fluidization, where the system experiences a competition of length scales: the amplitude  $a$ , the particle diameter  $d$ , and additionally the wavelength of the undulations  $\lambda$ . This becomes especially clear in the phase diagram of Fig. 14(a) for  $a = 2.0$  mm where the competition results in an alternation of states. By increasing  $a$  in Fig. 14(b,c) it becomes the dominant length scale and the alternation vanishes ultimately.

How does Fig. 14 compare with other phase diagrams for shaken granular matter in the literature?

First we discuss the experimental phase diagram by Wassgren *et al.* [22] for a bed of 1.28 mm glass beads at mild fluidization ( $\Gamma \leq 10$ ). For increasing  $\Gamma$ , they observe a series of transitions from a solid bed to undulations ("arching") in qualitative agreement with our own experiments at mild shaking. Their series of transitions is interlaced however with several phenomena (Faraday heaping, surface waves) that are not observed in our system. This is presumably due to the larger depth of their container (12.5 particle diameters, versus 5 in our container, which means that their setup deviates considerably from 2-D) and to the fact that their bed height was typically an order of magnitude larger than ours: The lowest aspect ratio  $L/h_0$  in their experiments was 2, versus 10 in our system. Hsiao and Pan [81], who conducted experiments in a similar setup in the mild fluidization regime, found the same sequence of phenomena as Wassgren *et al.* [22]. Indeed, in three dimensions a much wider variety of phenomena is observed than in 2-D systems [4, 5, 55, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98].

Secondly, Sunthar and Kumaran [59] construct a phase diagram (shaking strength vs. number of layers) based on

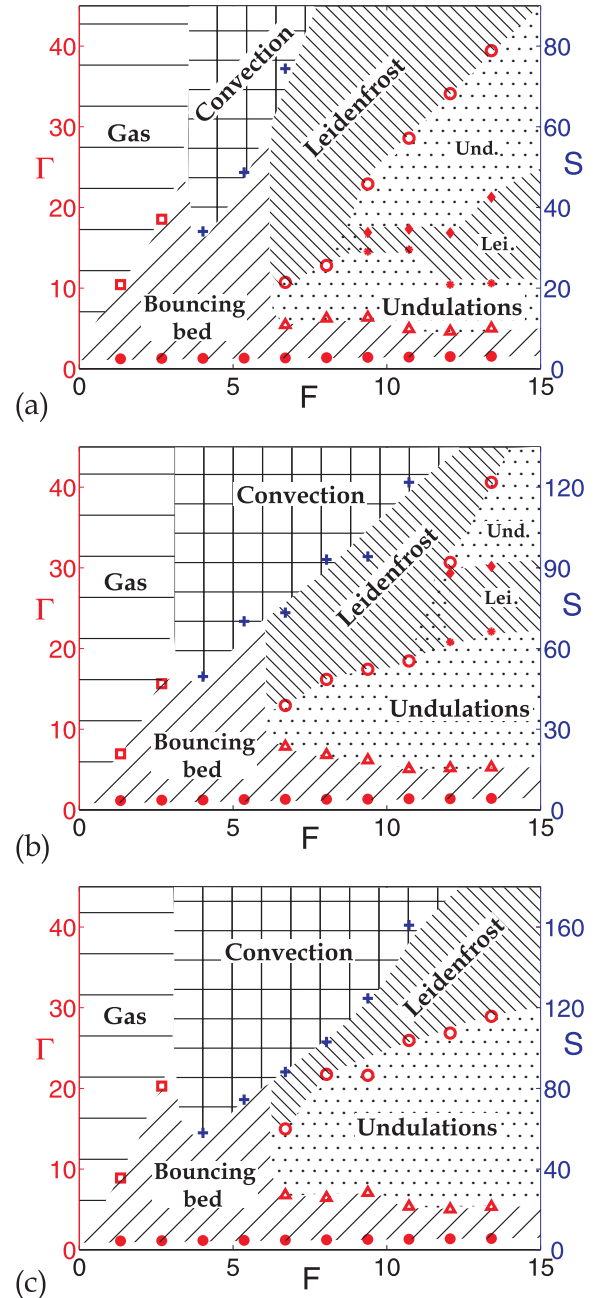


FIG. 14: (color online). Phase diagram of the shallow granular bed at three fixed values of the shaking amplitude: (a)  $a = 2.0$  mm, (b)  $a = 3.0$  mm, and (c)  $a = 4.0$  mm. The five phenomena studied in this paper are indicated by the different shadings. The onset values for bouncing bed, undulations, Leidenfrost effect, and gas are governed by the shaking parameter  $\Gamma$  (left vertical axis, red); the onset of convection is controlled by  $S$  (right vertical axis, blue). The narrow region without shading along the horizontal  $F$ -axis (below the bouncing bed regime) corresponds to the solid phase, in which the bed never detaches from the vibrating bottom. [Enhanced online: link to movie showing all transitions of diagram (b), for increasing shaking frequency at  $F = 8.1$  layers.]

event driven simulations in a 2-D system with an aspect ratio  $L/h_0 \gtrsim 10$ , comparable to ours. At low shaking strength, their phase diagram shows a region where the bed is "homogeneous", corresponding to the solid and bouncing bed regimes in our diagram. At higher shaking strength, they find a granular gas for  $F < 5$  and a region of granular convection for  $F > 5$ . The gaseous region compares well with the gas region in Fig. 14. The convection observed by Sunthar and Kumaran, however, occurs at a much milder fluidization than in our system. In contrast to our convection rolls, the density of their rolls is almost constant, indicating that the bed behaves more like a fluid than a gas.

Thirdly, Eshuis *et al.* [6] construct an experimental phase diagram (supported by a theoretical model) for a bed of 4 mm glass beads in a 2-D setup. The  $(S, F)$ -diagram shows a bouncing solid regime for low shaking strength and a gas region for small  $F$ . Between these two phases, for  $S \gtrsim 16$  and  $F > 8$ , the Leidenfrost regime is located. This is qualitatively the same as in Fig. 14, without the regions of undulations and granular convection though. The fact that these latter phenomena were absent is probably due to the much smaller aspect ratio ( $L/h_0 \sim 1$ ) and the much stronger confinement to two dimensions, since the depth of the container was just slightly more than one particle diameter.

Finally, Paolotti *et al.* [7] performed a 2-D numerical study of a granular bed with aspect ratio  $L/h_0 \approx 8$ , focusing on the transition towards convection. Their convection rolls show similar arches and distinct density differences as observed in our experiments. Starting from strong fluidization, for a given number of layers, they observe two transitions as the shaking strength is reduced. First a transition from a non-convective state (presumably a granular gas) to convection, followed by a transition towards a non-convective state again, in which the particles remain localized near the bottom. This latter state is not further specified, but most probably corresponds to a bouncing bed. In the phase diagram of Fig. 14 the same sequence is found if one follows a path from the gas regime to the bouncing bed via convection.

In conclusion, we have constructed the experimental phase diagram for a vertically shaken shallow granular bed in a quasi 2-D container, identifying the dimensionless control parameters that govern the various transitions in this diagram. In the present work we have concentrated on  $\Gamma$  and  $S$  (the shaking parameters), and the parameter  $F$  (number of particle layers). From the discussion above, it may be concluded that also the aspect ratio is an important control parameter, determining e.g. the set of different phenomena that a given system is able to exhibit.

The diagram of Fig. 14 shows the full range of phases that granular matter can display, behaving either like a solid, a fluid, or a gas [99, 100, 101, 102, 103]. A determination of the dimensionless parameters that govern the transitions between these phases is a crucial step towards

a better understanding of the physics of vertically shaken granular matter.

*Acknowledgment:* We thank Stefan Luding for stimulating discussions. This work is part of the research program of FOM, which is financially supported by NWO.



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